Math 1483 - Functions and Modeling
Exam 1 Solutions

---

1. The ideal gas law is an equation which is a reasonable approximation to the behaviour of some gases under a range of conditions. It was first stated by Benoît Paul Émile Clapeyron in 1834 and is given by the equation:

$PV = nRT$

Here, P stands for pressure, which is measured in atmospheres (atm), V is volume (in L), n is the quantity of gas moles (mol), R is the gas constant, which is specific to each gas (units in L·atm/mol·K), and T is temperature using the Kelvin scale (units K). If we wish to know the pressure (P) of the gas given all the other variables, we get the function:

$$P(n, R, T, V) = \frac{nRT}{V}$$

(a) How many variables is P a function of?

P is a function of four variables: n, R, T and V.

(b) What does $P(0.7, 0.09, 270, 3.04)$ represent? State your answer in a complete sentence.

$P(0.7, 0.09, 270, 3.04)$ represents the pressure of a 0.7 moles of a gas with gas constant 0.09 L·atm/mol·K, at a temperature of 270 K and volume 3.04 litres.

(c) Evaluate $P(0.7, 0.09, 270, 3.04)$ and explain what it means in a full sentence.

Rounding to two decimal places, $P(0.7, 0.09, 270, 3.04) = 5.60$ atm, which is the pressure of the gas given the values of n, R, T, and V as defined in part (b).

(d) If the gas is quickly transferred to a container ten times as large as the original container from parts (b) and (c), what is the resulting pressure?

We would need to evaluate $P(0.7, 0.09, 270, 30.4)$, which is simply one tenth the pressure we calculated in (c), e.g. $P(0.7, 0.09, 270, 30.4) = 0.56$ atm.

2. Figure 1 shows the elevation (in feet above sealevel) of Lake Texoma for the year 2015, the year of the floods. The blue graph is the actual elevation of the lake, the red is the desired elevation and the green graph you can is the level of the conservation pool. Use this figure to answer parts (a)–(f) below.

(a) Give a rough estimate on the days of the year over which the elevation on Lake Texoma was rising.

The elevation was rising from the beginning of January until June 1st, the middle of June to early-July, briefly in the middle of July, the end of November to very beginning of December, and then close to the end of December until December 31st.

(b) Give a rough estimate on the days of the year over which the elevation on Lake Texoma was decreasing.

Since the lake elevation never appeared constant, it was decreasing in elevation at all the times of the year not given in part (a).

(c) When was Lake Texoma at its highest level?

It appears that June 1st was the highest level, closely followed by another high mark in the third week of June.

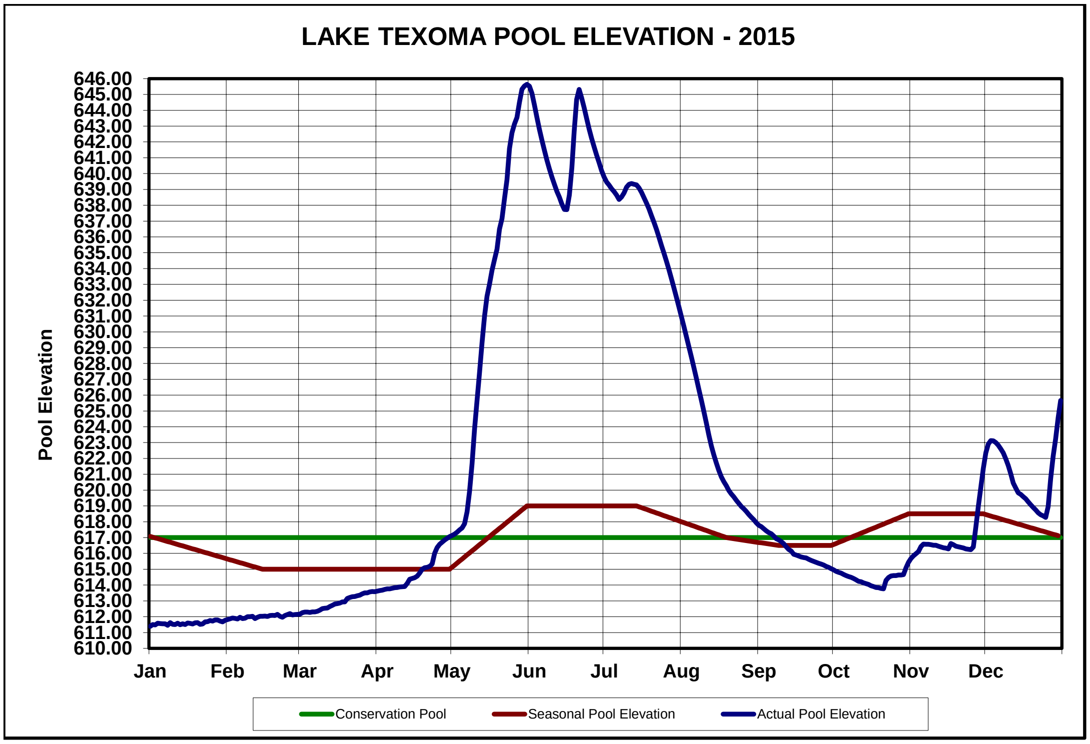

Figure 1: Lake Texoma pool elevation level for the year 2015.

(d) When was Lake Texoma at its lowest level?

The lake was at its lowest level on January 1st.

(e) When was Lake Texoma below its seasonal pool elevation?

This situation occurs when the blue graph is below the red graph, which is January 1st to the third week of April, and then again from approximately the second week of September until right before the end of November.

(f) When was Lake Texoma rising in elevation the fastest?

We are looking for where the graph is going up to the right the fastest, which appears to be either the middle of May, or right after the dip in the middle of June before the second peak.

3. The following is a table of the population of Durant from 1990 to 2015, given in 5 year increments:

year	population
1990	13,110
1995	13,050
2000	14,765
2005	15,316
2010	15,927
2015	17,214

(a) For what range of 5 years did Durant have the greatest average rate of change?

For the 5 years 1990 through 1995: $(13,050 - 13,110)/5 = -12$
For the 5 years 1995 through 2000: $(14,765 - 13,050)/5 = 343$
For the 5 years 2000 through 2005: $(15,316 - 14,765)/5 = 110.2$
For the 5 years 2005 through 2010: $(15,927 - 15,316)/5 = 122.2$
For the 5 years 2010 through 2015: $(17,214 - 15,927)/5 = 257.4$

So the 1995-200 time frame was when Durant's population had the greatest rate of change.

(b) What are the units on the average rates of change you calculated in part (a)?

The units are in people/year.

(c) Use the average rate of change to approximate the population of Durant in 2002.

The population in 2002 would be the population in 2000 plus twice the average rate of change calculated from 2000 to 2005, which gives $14,765 + 2 \cdot 110.2 = 14,985.4$, which we will round down to 14,985.

(d) Use the average rate of change from 2010 to 2015 to approximate the population of Durant in 2018.

We cannot use information that we do not have, namely the population in 2020, so we do as the problem asks us, and use the average rate of change from 2010 to 2015 instead. So the population in 2018 can be approximated as the population in 2015 plus 3 times the average rate of change, which gives $17,214 + 3 \cdot 257.4 = 17,986.2$ which we round down to 17,986.

(e) The recorded population of Durant in 2018 was 18,175. Compare this value to the value from part (d). What does this tell you about the rate of growth of Durant?

Since the actual population in Durant in 2018 was greater than the approximation which used the average rate of change from 2010 to 2015, we should expect that the rate of growth in Durant is increasing from 2015 to 2018.

4. In this problem, we will estimate the cost of a semester of living on campus at Southeastern. The cost per credit hour for an undergraduate face-to-face class is \$208.00 and for an online course is \$258.00. Every credit hour also incurs a \$107.00 in 'mandatory' fees of one kind or another. You have also decided to live by yourself in the North Halls, with your own private bathroom, which costs \$2155.00 per semester. For a meal plan, you chose Flex Choice 7 which runs you another \$1,650.00 per semester.

(a) Write an expression which relates the total cost T to the number of face-to-face credit hours (f) you wish to take and number of online credit hours (n).

There are many ways to write this, but they algebra will always work out to the same expression:

$$T(f, n) = \$208.00 \cdot f + \$258.00 \cdot n + \$107.00(f + n) + \$2155.00 + \$1,650.00$$

or simplifying and combining terms:

$T(f, n) = \$315.00 \cdot f + \$365.00 \cdot n + \$3805.00.$

(b) You have to take 3 face-to-face classes which are 3 credits, each, one face-to-face class which is 4 credits, and an online course with is 2 credits. Express this in function notation.

$T(13, 2).$

(c) Evaluate your answer to part (b).

$T(13, 3) = \$315.00 \cdot 13 + \$365.00 \cdot 2 + \$3805.00 = \$8630.00.$

(d) How much money would you save if you switched your online class to a face-to-face class?

One can simply compute $T(15, 0) - T(13, 2)$, or we can simply note that the difference in price (per credit hour) is \$50.00, so one would save \$100.00.