Math 1483 - Functions and Modeling
Exam 2

Assigned: 2020.10.13, 12:01 AM

Due: 2020.10.13 at 11:59 PM

Instructions: Work on this by yourself, if you feel you need to ask a question for clarification purposes, you may email the instructor. You are allowed to use your book, notes, and a graphing calculator/program. For each problem be sure to show all of your work and write every step down in a clear and concise manner. When finished, upload this front sheet and all of your work, as a pdf or jpg to Blackboard. Do not write your answers on the exam itself.

Agreement: Please read the following statement and then write it above the 'Printed Name' line:

"*I hereby swear that all the work that appears on this exam is completely my own, and I have not discussed any portion of this exam with any one else besides the instructor.*"

Printed Name: ______________________________

Signature: ______________________________

Date: ______________________________

1. The formula for conversion from degrees Celsius to degrees Fahrenheit is given by: $F(C) = 1.8C + 32$, where C is the temperature in Celsius, and F the temperature in Fahrenheit.

(a) Graph $F(C)$ for $-50 \leq C \leq 50$.

(b) Draw both the graph of $F(C)$ and $F = C$ on the same graph.

(c) Solve the equation $C = 1.8C + 32$ for C.

(d) How does your answer to part (c) relate to your answer from part (b)?

2. The formula to determine wind chill temperature used in Canada is given by

$$T_{wc} = 13.12 + 0.6215T_a - 11.37v^{0.16} + 0.3965T_a v^{0.16}.$$

Here T_a is the ambient temperature, in degrees Celsius, and v is the wind speed, in kilometers per hour, while T_{wc} is the resulting temperature, including wind chill, also measured in degrees Celsius.

In Australia, assuming a relative humidity of 30%, the wind chill temperature formula is given by

$$T_{wa} = T_a - 0.195v + 0.604e^{(17.27T_a)/(237.7+T_a)} - 4.$$

Similar to the definition of T_{wc}, T_a and T_{wa} are measured in degrees Celsius, and v is wind speed in kilometers per hour and is required to be at least 3 kph for this formula to hold.

(a) Explain what $T_{wc}(-10, 25)$ means, and then evaluate.

(b) Graph $T_{wc}(T_a, 10)$ and $T_{wa}(T_a, 10)$ for $-50 \leq T_a \leq 50$.

(c) From your graph in part (b), for what ambient temperatures do the wind chill models agree?

(d) Over what ambient temperature range does the Canadian model yield a higher temperature?

3. This is a continuation of problem 2, using the same formulas for T_{wc} and T_{wa}.

(a) Explain what $T_{wa}(-15, v)$ represents.

(b) Graph $T_{wc}(-15, v)$ and $T_{wa}(-15, v)$ for $3 \leq v \leq 65$ kph.

(c) Explain the behaviour of the graph, for both models, as v increases. Does it make sense?

(d) Which model appears to result in a colder wind chill temperature for moderate wind speeds if the ambient temperature is -15 C.

(e) At what two wind speeds do the models agree for $3 \leq v \leq 65$ kph?

4. This is a continuation of problems 2 and 3, using the same formulas for T_{wc} and T_{wa}.

(a) Graph $T_{wc}(0, v)$ and $T_{wa}(0, v)$ for $3 \leq v \leq 65$ kph.

(b) Do the two models ever agree on the same wind chill temperature for $3 \leq v \leq 65$ kph when $T_a = 0$ C?

exam continued on next page...

5. Wind chill appears to affect the cheeks of an uncovered individual more than another other part of the face when exposed to cold air and wind. The heat supplied to the surface of a face is limited by what is known as thermal resistivity, which depends on many factors – some of which involve the skin, underlying tissues, tissue thickness, and blood flow. Thermal resistivity is measured in m^2 K/W, and experiments have been conducted to measure thermal resistivity as a function of temperature. One such model is given by:

$$R(T) = 0.0389758 + 0.00554678T - 0.000286671T^2 + 3.39756 \times 10^{-6} T^3$$

where T is the windchill temperature measured in degrees Celsius, and R is the thermal resistivity measured in m^2 K/W.

(a) Graph $R(T)$ for temperatures in the range $0 \leq T \leq 40$ C.

(b) At what temperature does thermal resistance reach a maximum, and what is the value?