Math 1483 - Functions and Modeling

Exam 2 Solutions

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1. The formula for conversion from degrees Celsius to degrees Fahrenheit is given by: $F(C) = 1.8C + 32$, where C is the temperature in Celsius, and F the temperature in Fahrenheit.

(a) Graph $F(C)$ for $-50 \leq C \leq 50$.

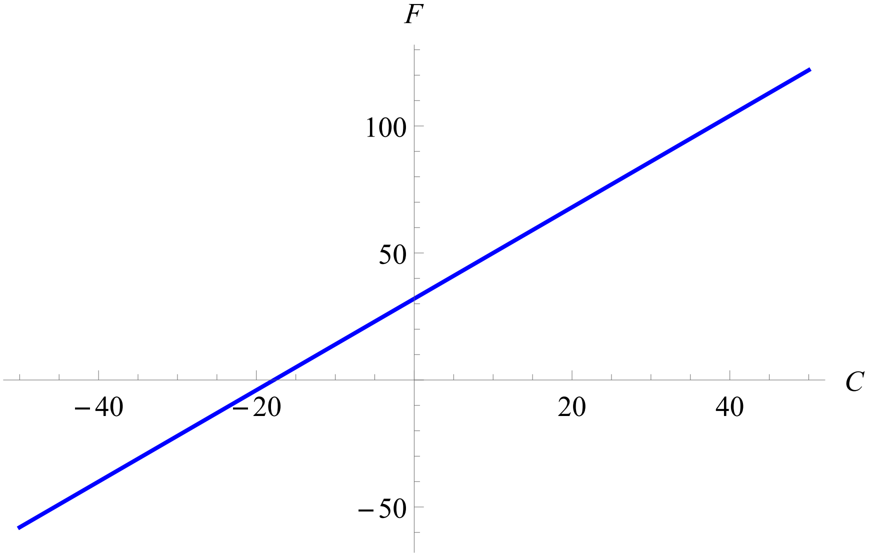

Figure 1: Graph of $F(C)$

(b) Draw both the graph of $F(C)$ and $F = C$ on the same graph.

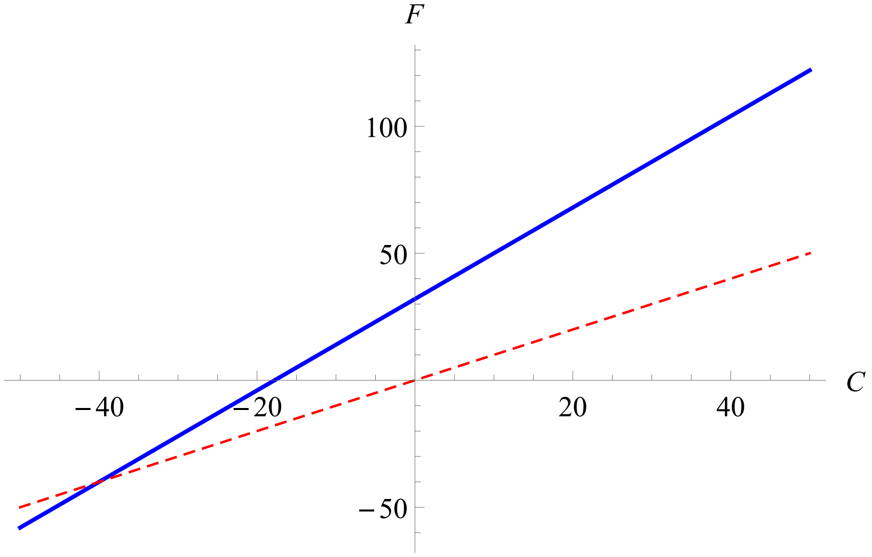

Figure 2: The point where $F = C$ is the intersection of the two lines.

(c) Solve the equation $C = 1.8C + 32$ for C.

Subtract $1.8C$ from both sides to get $-0.8C = 32$, then dividing both sides by -0.8 gives $C = -40$.

(d) How does your answer to part (c) relate to your answer from part (b)?

The point on the graph where $F = C$ is where the line $F = C$ intersects the line $F = .8C + 32$, which appears to be at $F = -40$ as was found in part (c).

2. The formula to determine wind chill temperature used in Canada is given by

$$T_{wc} = 13.12 + 0.6215T_a - 11.37v^{0.16} + 0.3965T_a v^{0.16}.$$

Here T_a is the ambient temperature, in degrees Celsius, and v is the wind speed, in kilometers per hour, while T_{wc} is the resulting temperature, including wind chill, also measured in degrees Celsius.

In Australia, assuming a relative humidity of 30%, the wind chill temperature formula is given by

$$T_{wa} = T_a - 0.195v + 0.604e^{(17.27T_a)/(237.7+T_a)} - 4.$$

Similar to the definition of T_{wc}, T_a and T_{wa} are measured in degrees Celsius, and v is wind speed in kilometers per hour and is required to be at least 3 kph for this formula to hold.

(a) Explain what $T_{wc}(-10, 25)$ means, and then evaluate.

$T_{wc}(-10, 25) = -18.76$ C is the wind chill temperature given the ambient temperature is -10 C, and the wind speed is 25 kph, using the Canadian model.

(b) Graph $T_{wc}(T_a, 10)$ and $T_{wa}(T_a, 10)$ for $-50 \le T_a \le 50$.

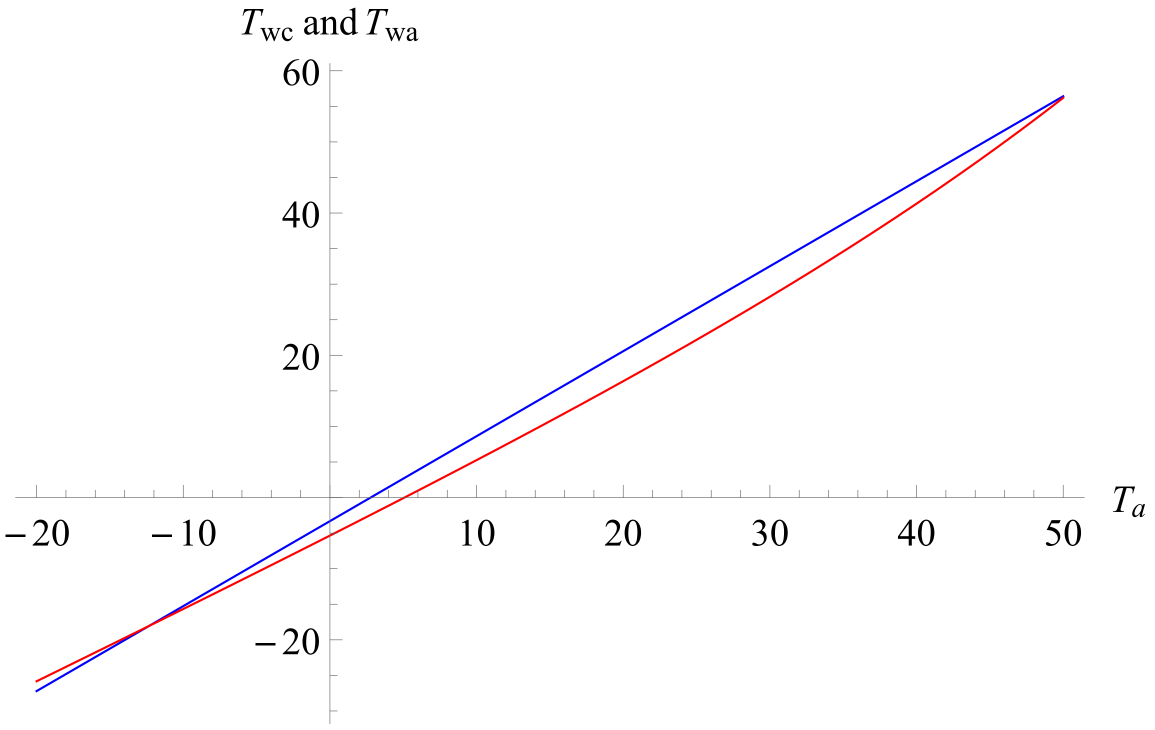

Figure 3: Graphs of T_{wc} (blue) and T_{wa} (red) when $v = 10$ kph.

(c) From your graph in part (b), for what ambient temperatures do the wind chill models agree?

Upon inspection of the graph, it appears that $T_a \approx -12.3$ C and $T_a \approx 50.5$ C are the two temperatures at which the models agree.

(d) Over what ambient temperature range does the Canadian model yield a higher temperature?

The range is the interval between the two points found in part (d), so $-12.3 \leq T_a \leq 50.5$ C is the range.

3. This is a continuation of problem 2, using the same formulas for T_{wc} and T_{wa}.

(a) Explain what $T_{wa}(-15, v)$ represents.

$T_{wa}(-15, v)$ is the wind chill temperature, in degrees Celsius, when ambient temperature is -15 C, for an arbitrary velocity v, using the Australian model.

(b) Graph $T_{wc}(-15, v)$ and $T_{wa}(-15, v)$ for $3 \leq v \leq 65$ kph.

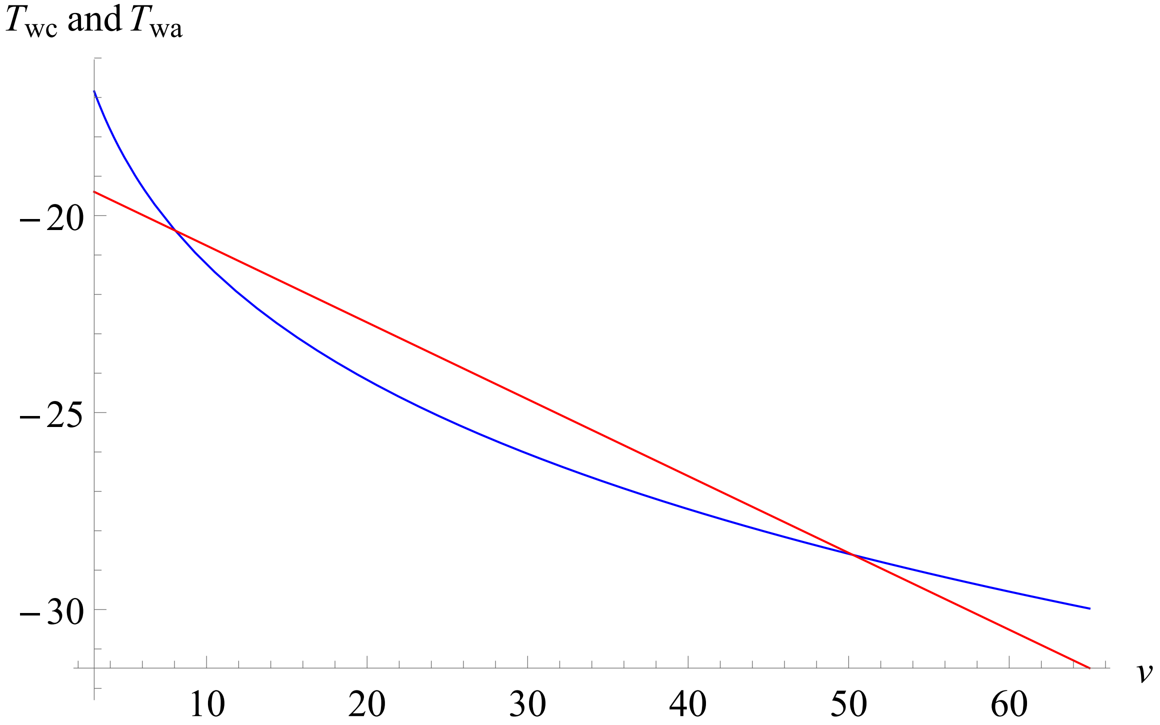

Figure 4: Graphs of T_{wc} (blue) and T_{wa} (red) when $T_a = -15$ C.

(c) Explain the behaviour of the graph, for both models, as v increases. Does it make sense?

The output, the wind chill for an ambient temperature of -15 C decreases as wind speed increases. We know this to be true, the higher the wind speed, the colder it feels.

(d) Which model appears to result in a colder wind chill temperature for moderate wind speeds if the ambient temperature is -15 C.

The T_{wa} model lies below the T_{wc} model for a large portion of the graph, so T_{wa} results in colder wind chill temperatures.

(e) At what two wind speeds do the models agree for $3 \leq v \leq 65$ kph?

The intersection points appear to be $v = 8.0$ kph and $v = 50.3$ kph.

4. This is a continuation of problems 2 and 3, using the same formulas for T_{wc} and T_{wa}.

(a) Graph $T_{wc}(0, v)$ and $T_{wa}(0, v)$ for $3 \leq v \leq 65$ kph.

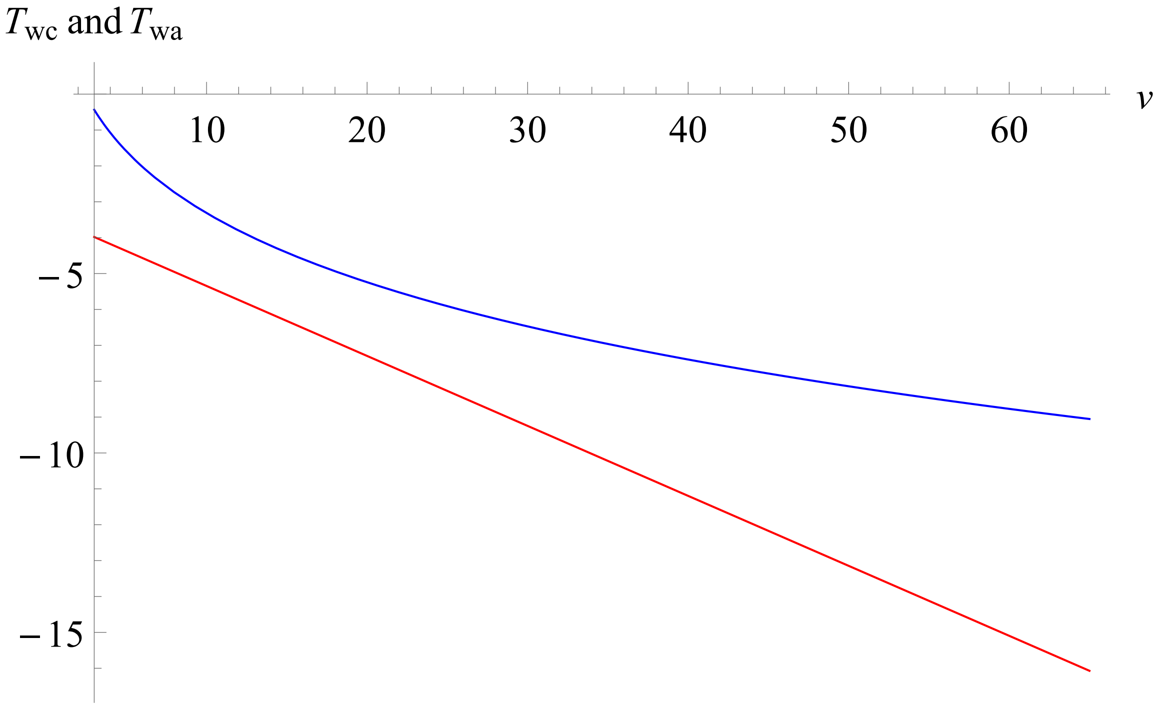

Figure 5: Graphs of T_{wc} (blue) and T_{wa} (red) when $T_a = 0$ C.

(b) Do the two models ever agree on the same wind chill temperature for $3 \leq v \leq 65$ kph when $T_a = 0$ C?

No, in fact, $T_{wc} > T_{wa}$ for all wind speeds under consideration.

5. Wind chill appears to affect the cheeks of an uncovered individual more than another other part of the face when exposed to cold air and wind. The heat supplied to the surface of a face is limited by what is known as thermal resistivity, which depends on many factors – some of which involve the skin, underlying tissues, tissue thickness, and blood flow. Thermal resistivity is measured in m^2 K/W, and experiments have been conducted to measure thermal resistivity as a function of temperature. One such model is given by:

$$R(T) = 0.0389758 + 0.00554678T - 0.000286671T^2 + 3.39756 \times 10^{-6}T^3$$

where T is the windchill temperature measured in degrees Celsius, and R is the thermal resistivity measured in m^2 K/W.

(a) Graph $R(T)$ for temperatures in the range $0 \leq T \leq 40$ C.

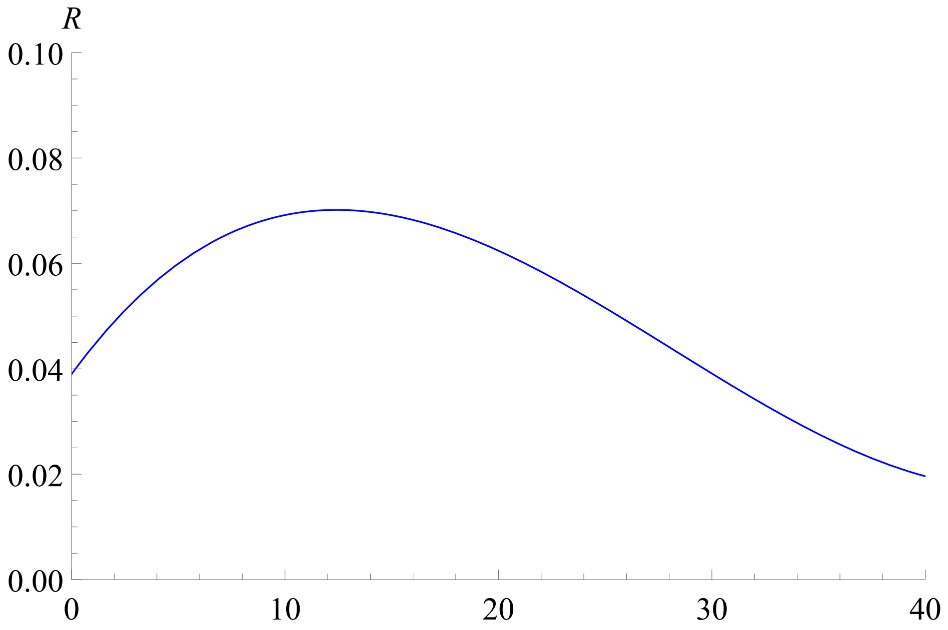

Figure 6: Thermal resistivity as a function of wind chill temperature in degrees Celsius.

(b) At what temperature does thermal resistance reach a maximum, and what is the value?

Thermal resistance is maximized at approximately $T = 12.4$ C, with a value of $R = 0.07$ m^2 K/W.