Math 1513 - College Algebra
Exam 1 Solutions

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1. Simplify the following expression into an expression with no denominator which is the product of four terms.

$$\frac{\sqrt[3]{2}x^7y^{1/4}z^{-2/3}}{\sqrt[5]{2}\sqrt{x}y^3\sqrt[4]{z^{-3}}}$$

We move all the powers in the denominator into the numerator:

$$\frac{\sqrt[3]{2}x^7y^{1/4}z^{-2/3}}{\sqrt[5]{2}\sqrt{x}y^3\sqrt[4]{z^3}} = 2^{1/3}\cdot 2^{-1/5}\cdot x^7\cdot x^{-1/2}\cdot y^{1/4}\cdot y^{-3}\cdot z^{-2/3}z^{3/4}$$

Then we add exponents for terms with like bases:

$$2^{1/3}\cdot 2^{-1/5}\cdot x^7\cdot x^{-1/2}\cdot y^{1/4}\cdot y^{-3}\cdot z^{-2/3}\cdot z^{-2/3}z^{3/4} = 2^{1/3-1/5}\cdot x^{7-1/2}\cdot y^{1/4-3}\cdot z^{-2/3+3/4}$$

Lastly we simplify the fractions in each exponent:

$$2^{1/3-1/5}\cdot x^{7-1/2}\cdot y^{1/4-3}\cdot z^{-2/3+3/4} = 2^{2/15}\cdot x^{13/2}\cdot y^{-11/4}\cdot z^{1/12}$$

2. Expand and simplify, writing your answer in decreasing powers of x:

$$(\sqrt[3]{5}+1-\sqrt{x})(6x+\sqrt[3]{5^2}-2)$$

We foil, expecting 9 terms total:

$$\begin{aligned}(\sqrt[3]{5}+1-\sqrt{x})(6x+\sqrt[3]{5^2}-2) &= 6\sqrt[3]{5}x+\sqrt[3]{5}\sqrt[3]{5^2}-2\sqrt[3]{5}+6x+\sqrt[3]{5^2}-2-6x\sqrt{x}-\sqrt[3]{5^2}\sqrt{x}+2\sqrt{x}\\ &= 6\sqrt[3]{5}x+5-2\sqrt[3]{5}+6x+\sqrt[3]{5^2}-2-6x\sqrt{x}-\sqrt[3]{5^2}\sqrt{x}+2\sqrt{x}\\ &= -6x^{3/2}+(6\sqrt[3]{5}+6)x+(2-\sqrt[3]{5^2})\sqrt{x}+3-2\sqrt[3]{5}+\sqrt[3]{5^2}\end{aligned}$$

3. Find a common denominator, simplify, and factor fully:

$$\frac{3m}{5-3m}-\frac{1}{2m-1}+\frac{2+17m}{(5-3m)(2m-1)}$$

The common denominator is $(5-3m)(2m-1)$:

$$\frac{3m}{5-3m}-\frac{1}{2m-1}+\frac{2+17m}{(5-3m)(2m-1)} = \frac{2m-1}{2m-1}\cdot\frac{3m}{5-3m}-\frac{5-3m}{5-3m}\cdot\frac{1}{2m-1}+\frac{2+17m}{(5-3m)(2m-1)}$$

Putting this all over one fraction gives:

$$\frac{2m-1}{2m-1}\cdot\frac{3m}{5-3m}-\frac{5-3m}{5-3m}\cdot\frac{1}{2m-1}+\frac{2+17m}{(5-3m)(2m-1)} = \frac{(2m-1)\cdot 3m-(5-3m)+(2+17m)}{(5-3m)(2m-1)}$$

Combining like terms gives:

$$\frac{(2m-1)\cdot 3m-(5-3m)+(2+17m)}{(5-3m)(2m-1)} = \frac{6m^2+17m-3}{(5-3m)(2m-1)}$$

Lastly, the numerator factors ‘nicely’, so our final answer is:

$$\frac{6m^2+17m-3}{(5-3m)(2m-1)} = \frac{(3+m)(6m-1)}{(5-3m)(2m-1)}$$

4. Determine the allowable values for the following expression (assuming we are looking for real answers):

$\sqrt{\dfrac{x+1}{x-1}}$

Remember that for real results, we require the argument of the square root to be non-negative. This, we are solving:

$$\frac{x+1}{x-1} \geq 0$$

Note however, that the denominator involves the variable x, so we cannot simply multiply both sides by $x-1$. However, if $x-1>0$, then we can. This requires $x>1$. So if $x>1$, solving $x+1 \geq 0$ gives $x \geq -1$. This requires us to choose $x>1$, which is the intersection of the two intervals. If $x-1<0$ (or $x<1$), then we want $x+1 \leq 0$, which is $x \leq -1$. Thus, here the intersection of the two intervals is $x \leq -1$. So our final answer is the union of the two solution intervals: $x \in (-\infty, -1] \cup (1, \infty)$.

5. Solve for x in the equation: $3x - 2 = 4x + 7$.

We simply isolate x by subtracting $3x + 7$ from both sides, which gives $-9 = x$, or $x = -9$.

6. For what value(s) of b does the following equation have *no* solution: $\frac{1}{3}x - 4 = bx + 2$.

If we solve for x, we get: $x = \dfrac{6}{1/3 - b}$. So if $b = 1/3$, the denominator is zero, which is equivalent to starting with the equation $-4 = 2$, which is never true for any x.

7. For what value(s) of b does the following equation have an *infinite* number of solutions: $4x + 1 = 2(2x + b)$.

Expanding the above equation gives $4x + 1 = 4x + 2b$. If $b = 1/2$, then the equation becomes $4x + 1 = 4x + 1$, which is true for any x.

8. Solve for all solutions to the following equation by factoring: $x^5 - 9x^4 = 36x^3$

First note that we can move everything over to the left hand side, and factor an x^3 out: $x^3(x^2 - 9x - 36) = 0$. Note that $x^2 - 9x - 36$ can be factored as $(x-12)(x+3)$, so our fully factored form for the equation is: $x^3(x-12)(x+3) = 0$. By the product of zeros rule, this gives solutions $x = 0, 12, -3$.

9. Solve for x in the following equation: $5 - 3x = \sqrt{x+5} + 6$.

We will need to get rid of the square root to solve for x, so first we get it by itself on the right hand side by subtracting 6 from both sides:

$$-1 - 3x = \sqrt{x+5}$$

Now we square both sides:

$$1 + 6x + 9x^2 = x + 5$$

Moving all terms to one side:

$$9x^2 + 5x - 4 = 0$$

Which factors as

$$(x+1)(9x-4) = 0$$

So our solutions are $x = -1$ and $x = 4/9$. If we substitute those both back into the original equation, we get $2 = 2$ for $x = -1$. For $x = 4/9$, we get $-7/3 = 7/3$, which is false. So $x = 4/9$ is an extraneous solution, and our only true solution is $x = -1$.

10. Factor and solve for x completely in the following equation, your answers *may* include complex numbers: $x^8 - 17x^4 + 16 = 0$.

If we let $u = x^4$, this equation becomes $u^2 - 17u + 16 = 0$, which factors as $(u-16)(u-1) = 0$, and thus, in terms of the variable x, we have $(x^4 - 16)(x^4 - 1) = 0$. But $x^4 - 16$ and $x^4 - 1$ are both differences of perfect squares, and we therefore get $x^4 - 16 = (x^2 - 4)(x^2 + 4)$, and similarly, $x^4 - 1 = (x^2 - 1)(x^2 + 1)$. So far, this gives:

$$(x^2 - 1)(x^2 + 1)(x^2 - 4)(x^2 + 4) = 0$$

But $x^2 - 1 = (x-1)(x+1)$, and $x^2 - 4 = (x-2)(x+2)$. The expression $x^2 + 1$ has complex roots, since setting it to zero gives: $x^2 = -1$, so $x^2 + = (x-i)(x+i)$. Similarly, $x^2 + 4 = (x-2i)(x+2i)$. We now have factored everything into linear terms:

$$(x-1)(x+1)(x-i)(x+i)(x-2)(x+2)(x-2i)(x+2i) = 0$$

The solutions are thus $x = \pm 1, \pm 2, \pm i \pm 2i$.

11. Solve for x in the following inequality. Express your answer in interval notation.

$$\frac{1}{3} < \left|2x + \frac{1}{7}\right| \leq \frac{5}{6}$$

We solve two inequalities, and then union their answers:

$$\frac{1}{3} < 2x + \frac{1}{7} \leq \frac{5}{6} \quad \text{or} \quad \frac{1}{3} < -\left(2x + \frac{1}{7}\right) \leq \frac{5}{6}$$

For the left inequality, we subtract 1/7 from both sides first:

$$\frac{4}{21} < 2x \leq \frac{29}{42}$$

and then divide by 2 to solve for x:

$$\frac{2}{21} < x \leq \frac{29}{84}.$$

For the right inequality in the *or* statement, we start by multiplying all sides by -1 and switching the direction of the inequalities:

$$-\frac{5}{6} \leq 2x + \frac{1}{7} < -\frac{1}{3}$$

Subtracting 1/7 from all three sides gives:

$$-\frac{41}{42} \leq 2x < -\frac{10}{21}$$

Diving all sides by 2 once again allows us to solve for x:

$$-\frac{41}{84} \leq x < -\frac{5}{21}.$$

So our final solution is:

$$-\frac{41}{84} \leq x < -\frac{5}{21} \quad \text{or} \quad \frac{2}{21} < x \leq \frac{29}{84}.$$

In interval notation, this is

$$x \in \left[-\frac{41}{84}, -\frac{5}{21}\right) \cup \left(\frac{2}{21}, \frac{29}{84}\right]$$

12. The absolute value of a complex number $z = a + bi$, which is denoted $|z|$ as is the case with real numbers, is given by the formula $|z| = \sqrt{a^2 + b^2}$. Use this definition to answer both parts of this problem.

(a) If $z = \sqrt{2} - \sqrt{3}i$, compute $|z|$.

$|z| = \sqrt{\sqrt{2}^2 + (-\sqrt{3})^2} = \sqrt{2+3} = \sqrt{5}$

(b) Find 5 complex numbers $|z|$, none of which are purely real or purely imaginary, such that $|z| = 7$.

We need to find real numbers a and b such that $\sqrt{a^2 + b^2} = 7$, or $a^2 + b^2 = 49$. There are an infinite number of answers, but for instance, if we set $a = 1$, then $b^2 = 48$, or $b = \pm\sqrt{48}$, so two such numbers are $z = 1 \pm \sqrt{48}i$. Clearly then, $z = \sqrt{48} \pm i$ will also work. Setting $a = 2$, then $b^2 = 44$, or $b = \sqrt{44} = 2\sqrt{11}$. So two more numbers are $z = 2 \pm 2\sqrt{11}i$.