Math 2283 - Honors Logic

Homework - Chapter 4

Name: ______________________________

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1. Consider the following sets:

   $A = \{\, x \mid x$ is a US state bordering the Atlantic Ocean. $\}$

   $B = \{\, x \mid x$ is a US state bordering the Pacific Ocean. $\}$

   $C = \{\, x \mid x$ is a US state which has a coast on the Gulf of Mexico. $\}$

   $D = \{\, x \mid x$ is a US state which shares a border with Canada. $\}$

   $E = \{$ Texas, Oklahoma, Arkansas $\}$

   $F = \{\, x \mid x$ is a US state which shares no border with any other US state. $\}$

   $G = \{\, x \mid x$ is a US state whose has as part of its border the Red River of the South. $\}$

Relate the following pairs of sets using the relations: subset, superset, overlap, disjoint, and equal.

(a) $A \quad B$	(b) $A \quad C$	(c) $F \quad B$
(d) $E \quad G$	(e) $D \quad B$	(f) $C \quad G$

2. Determine which of the fundamental relations (cf. Theorem 4.1) hold between the following pairs of intervals:

(a) $[2,4],[5,8]$	(b) $[4,7],[3,5]$	(c) $[4,7],[4,7]$
(d) $[1,3],[-2,4]$	(e) $[1,3],[1,2]$	(f) $[0,1],[-2,1]$
(g) $[1,7],[-2,4]$	(h) $[-2,10],[1,2]$	(i) $[1,1],[-1,2]$

3. Draw two boxes K and L so that they stand in one of the following relations (cf. Theorem 4.1):
   (a) $K = L$
   (b) the box K is a proper subclass of the box L
   (c) the box L is a proper subclass of the box K
   (d) the boxes K and L overlap
   (e) the boxes K and L are disjoint

4. Which of the cases from problem 3 are eliminated if K and L are congruent?

5. Which of the cases from problem 3 are eliminated if we consider only the perimeters of K and L (hence K and L are rectangles)?

6. Is the following sentence (which has the same structure as the Law 4.4, the Law of Class Transitivity for Inclusion, of Section 4.4) true?

   If K is disjoint from L and L is disjoint from M, then K is disjoint from M.

7. Convert the following logical sentence into set notation so that there are no logical connectives or element symbols, instead only set operations and set relations.

$$\sim (x \in K \vee x \in L) \longleftrightarrow (\sim x \in K \wedge \sim x \in L)$$

8. Let $\triangle ABC$ be an arbitrary triangle, with an arbitrary point D lying on line segment $\overline{BC}$. Express your answers to the following two questions in formulas:
   (a) What figures are formed by the union of the two triangles $\triangle ABD$ and $\triangle ACD$?
   (b) What figures are formed by the intersection of the two triangles $\triangle ABD$ and $\triangle ACD$?

9. Represent an arbitrary square:

(a) as the union of two trapezoids,
(b) as the intersection of two triangles.

10. Let K and L be two arbitrary classes. What classes are $K \cup L$ and $K \cap L$ in the case $K \subseteq L$?

11. Let K be an arbitrary class. Determine each of the following classes:
(a) $K \cup U$ (b) $K \cap U$ (c) $\emptyset \cup K$ (d) $\emptyset \cap K$

Prove each of the following laws, given arbitrary classes K, L, and M.
12. **Law of Simplification for Union**: $K \subseteq K \cup L$
13. **Law of Simplification for Intersection**: $K \cap L \subseteq K$
14. **Distributive Law of Intersection over Union**: $K \cap (L \cup M) = (K \cap L) \cup (K \cap M)$
15. **Distributive Law of Union over Intersection**: $K \cup (L \cap M) = (K \cup L) \cap (K \cup M)$
16. **The Law of Double Complement**: $(K')' = K$
17. **De Morgan's Law for Union**: $(K \cup L)' = K' \cap L'$
18. **De Morgan's Law for Intersection**: $(K \cap L)' = K' \cup L'$

19. Consider the following three sets:
(a) The set of all natural numbers greater than 0 and less than 4.
(b) The set of all rational numbers greater than 0 and less than 4.
(c) The set of all irrational numbers greater than 0 and less than 4.

Which of these set are finite and which are infinite?

20. Define $U = \mathbb{N}$ to be the set of all positive integers, and $\mathbb{K} = \{6, 12, 18, \ldots\}$. Determine *all* values which make the following sentential function true.

$$\forall x \in \mathbb{K} \ (x/y \in \mathbb{N} \wedge y \in \mathbb{K}')$$