

Math 2143 - Brief Calculus with Applications

Final Exam - 2021.05.03

Name: _____

1. State the limit definition of the derivative of $f(x)$ at $x = a$.

2. State the limit definition of a function $f(x)$ being continuous at $x = a$.

3. Compute the following limits:

(a) $\lim_{x \rightarrow 2} 3x^2 - 4x + 1$

(b) $\lim_{x \rightarrow 2} \frac{(x-2)(2x+1)}{x+2}$

3. Compute the following limits (continued):

$$(c) \lim_{x \rightarrow 2} \frac{(x-2)(2x+1)}{x-2}$$

$$(d) \lim_{x \rightarrow 1^-} \frac{3x^2 + 2}{x - 1}$$

$$(e) \lim_{x \rightarrow 1^+} \frac{3x^2 + 2}{x - 1}$$

3. Compute the following limits (continued):

$$(f) \lim_{x \rightarrow \infty} \frac{3x + 2}{4x^2 - 4x + 1}$$

$$(g) \lim_{x \rightarrow -\infty} \frac{4x^2 - 2x + 1}{3x^2 + 5x + 2}$$

4. Complete the following derivative rule formulas:

$$(a) \frac{d}{dx} f(x) \cdot g(x)$$

$$(b) \frac{d}{dx} \frac{f(x)}{g(x)}$$

4. Complete the following derivative rule formulas (continued):

$$(c) \frac{d}{dx} f(g(x))$$

$$(d) \frac{d}{dx} f(x)^n$$

$$(e) \frac{d}{dx} \ln(f(x))$$

$$(f) \frac{d}{dx} e^{f(x)}$$

5. Compute the following derivatives, do not simplify your answer.

$$(a) \frac{d}{dw} \sqrt{3w^3 - \frac{1}{w} + 3}$$

5. Compute the following derivatives, do not simplify your answer (continued).

(b) $\frac{d}{dx} x^3 e^{3x^2} - 1$

(c) $\frac{d}{dy} \frac{4y + 2}{3y^2 + 1}$

(d) $\frac{d}{dz} z^3 e^{3z^2} - 1$

5. Compute the following derivatives, do not simplify your answer (continued).

(e) $\frac{d}{dt} \ln(4t^3 - 6x + 2) + t - 1$

6. The function $f(x) = \frac{1}{6}x^6 + \frac{2}{5}x^5 - \frac{1}{4}x^4 - \frac{2}{3}x^3 - 7$ has a derivative $f'(x)$ which can be factored as

$$f'(x) = x^2(x - 1)(x + 1)(x + 2)$$

(a) Determine the critical points of $f(x)$.

(b) State the intervals of increase and decrease of $f(x)$.

(c) Classify the critical points from part (a) as local max, mins, or neither using your work from part (b).

7. Find the absolute max and min of the function $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x$ on the interval $[-2, 3]$.

8. Use logarithmic differentiation to compute the derivative of the following function:

$$T(x) = \frac{(3x + 1)(5x - 7)^3(2 - 6x)^{3/2}}{\sqrt{8x + 1}(4x^2 + 7)}$$

9. Sketch the graph of the rational function $R(x) = \frac{3(x-2)^2}{(x-1)(x+2)}$ without taking derivatives. In order to do so, you should compute the following: (a) the domain, (b) roots, (c) y -intercept, (d) vertical asymptotes, and (e) horizontal asymptotes.