Math 4982 - Senior Seminar
Abstract Algebra Review Questions

1. Find the index of the subgroup generated by the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 3 & 1 & 2 & 4 & 5 \end{pmatrix}$$

in the alternating group A_5.

2. Let the set S be infinite and let the set T be countably infinite. Let $\overline{S}$ denote the complement of S. If S and T are both subsets of the real numbers, which of the following pairs of sets must be of the same cardinality?

3. How many topologies are possible on a set of 2 points?

4. Find the remainder on dividing 3^{20} by 7.

5. A subgroup H in group G is invariant if $gH = Hg$ for every $g \in G$. If H and K are both invariant subgroups of G, which of the following is also an invariant subgroup?

  (A) $H \cap K$ (B) HK (C) $H \cup K$ (D) Both (A) and (B)   (E) Both (B) and (C)

6. Let R be a ring such that $x^2 = x$ for all $x \in R$. Which of the following must be true?

  (A) $x = -x$ for all $x \in R$ (B) R is commutative   (C) $xy + yx = 0$ for all $x, y \in R$
  (D) Both (A) and (C)   (E) (A), (B), and (C)

7. In the integral domain $D = \{ r + s\sqrt{17} \mid r, s \in \mathbb{Z} \}$, which of the following is irreducible?

  (A) $8 + 2\sqrt{17}$ (B) $3 - \sqrt{17}$ (C) $9 - 2\sqrt{17}$ (D) $7 + \sqrt{17}$ (E) $13 + \sqrt{17}$

8. In the finite field $\mathbb{Z}_{17}$, the multiplicative inverse of 10 is

9. How many elements of D_{500} order 500?

10. If $\alpha = (1, 2, 3, 4)(5, 6, 7, 1, 8)$ what is α^{407}?

11. If G is the group of all rotations of the a cube, what is $|G|$?

12. How many group homomorphisms are there from $\mathbb{Z}_{15}$ to D_{77}?

13. If G has the following Cayley table, what must G be isomorphic to?

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
a_1	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
a_2	a_2	a_5	a_1	a_7	a_6	a_4	a_9	a_3	a_8
a_3	a_3	a_1	a_8	a_6	a_2	a_5	a_4	a_9	a_7
a_4	a_4	a_7	a_6	a_3	a_9	a_8	a_1	a_5	a_2
a_5	a_5	a_6	a_2	a_9	a_4	a_7	a_8	a_1	a_3
a_6	a_6	a_4	a_5	a_8	a_7	a_9	a_3	a_2	a_1
a_7	a_7	a_9	a_4	a_1	a_8	a_3	a_2	a_6	a_5
a_8	a_8	a_3	a_9	a_5	a_1	a_2	a_6	a_7	a_4
a_9	a_9	a_8	a_7	a_2	a_3	a_1	a_5	a_4	a_6

14. How many elements of S_6 have order 4?

15. If G is a group of order 35, how many subgroups of order 5 does G have? How many subgroups of order 7? Find a complete list (up to to isomorphism) of what G can be isomorphic to.

16. Up to isomorphism, how many Abelian groups of order 392 are there?

17. Suppose $n \in \mathbb{N}$ and up to isomorphism there are exactly 5 Abelian groups of order n. What does this tell you about n?

18. Let I be an ideal in the integers which contains 18 and 30. What are the possibilities for I?

19. How many nilpotent elements does $\mathbb{Z}_{72}$ have?

20. If today is Tuesday, what day of the week will it be in 10^{1100} days?

21. In the ring $\mathbb{Z}_5[x]$, what do the elements of the ideal $\langle x^2 + 6 \rangle$ and the factor ring $\mathbb{Z}_5[x]/\langle x^2 + 6 \rangle$ look like? (hint: polynomial long division works over any field) Determine if the ideal $\langle x^2 + 6 \rangle$ is maximal, prime, or neither.

22. Explain why the set $\{(x, x) | x \in \mathbb{Z}\}$ is a subring of $\mathbb{Z} \bigoplus \mathbb{Z}$ but not an ideal of $\mathbb{Z} \bigoplus \mathbb{Z}$.