

Math 4982 - Senior Seminar Calculus 1 Review Questions

1. Compute the following limit: $\lim_{x \rightarrow 0} \frac{\tan(x^2)}{4x^2}$
2. Compute the following limit: $\lim_{x \rightarrow -\infty} \frac{3e^{-x} - 4e^{2x}}{2e^{-x} + 5e^{3x}}$
3. Find an equation of the tangent line to $y = 3 \tan(2x) + 1$ at $x = \frac{\pi}{8}$.
4. Compute the following derivative: $\frac{d}{dz} \sin^2(\tanh^{-1}(2z))$
5. Compute the following derivative: $\frac{d}{dr} \int_{r^2}^{\sin(r)} e^{2t} + t^2 dt$
6. Find an equation of the tangent line to $y^2 + x \cos(y) = 4 - x$ at $(x, y) = (2, 0)$.
7. Sketch a graph of $f(x) = \frac{x}{\sqrt{x^2 + 2}}$. In order to accomplish this, compute and state (a) the domain, (b) x -, y -intercepts, (c) vertical and horizontal asymptotes, (d) critical points, (e) intervals of increase and decrease, (f) inflection points, and (g) intervals of concavity.
8. Find the point on the curve $y = \cos(x)$ closest to the origin.
9. A rectangular tank with base $4 \text{ m} \times 2 \text{ m}$ and height 3 m is being filled at a rate of $2 \text{ m}^3/\text{min}$. Determine the rate at which the height of the water in the tank is rising when the tank is half full.
10. Set up the integral(s), *but do not evaluate*, to compute the finite area bounded by the functions $y = 2x(3 - x)$, $y = -2x$, and $y = 4x - 12$.
11. Set up the integral(s), *but do not evaluate*, to compute the volume of surface of revolution of the region bounded by $y = \sin(x)$ and $y = \cos(x)$ for $\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$ and revolved around the line $x = -1$. You may use choose whichever method you wish for this problem.
12. Set up the integral(s), *but do not evaluate*, to compute the volume of surface of revolution of the region bounded by $y = \sin(x)$ and $y = \cos(x)$ for $\frac{\pi}{4} \leq x \leq \frac{5\pi}{4}$ and revolved around the line $y = 2$. You may use choose whichever method you wish for this problem.
13. Let $f(x) = \tanh^{-1}(\sin(x))$, $g(x) = \sinh^{-1}(\tan(x))$, and $h(x) = \cosh^{-1}(\sec(x))$. Compute the derivatives $f'(x)$, $g'(x)$, and $h'(x)$ and then *simplify as fully as possible*. Finally, make a clever observation.
14. Compute the following derivative: $\frac{d}{dx} f(g(x))^{a(b(x))}$
15. Compute the following definite integral: $\int_e^{e^2} \frac{1}{t \ln(t)} dt$
16. Compute the following integral: $\int \frac{x + 1}{\sqrt{1 - x^2}} dx$
17. Compute the following integral: $\int \frac{e^z}{1 + e^{2z}} dz$
18. Compute the following integral: $\int \frac{1}{\sqrt{25 + 4v^2}} dv$