Math 4982 - Senior Seminar
Linear Algebra Review Questions

Use the following system of equations to answer problems 1 – 5.

$$\begin{cases} 3x + 5y - 7z = 1 \\ 4x + 2y - 6z = 0 \\ 2x - 8y + 3z = -1 \end{cases}$$

1. Write the system of equations in $AX = B$ form.

2. Compute $\det(A)$.

3. Compute A^{-1}.

4. Compute $A^{-1}B$.

5. Write the solution to the system of equations using the previous problems.

Consider the set $\mathbf{K} = \{\langle 1, 2, 0, 1\rangle, \langle -1, 2, 1, 0\rangle\}$ for problems 6 – 10.

6. Compute the dimension of the vector space $\mathbb{S}$ spanned by $\mathbf{K}$.

7. Convert $\mathbf{K}$ to an orthonormal basis $\mathbf{B}$.

8. Compute a basis for $\mathbb{S}^\perp$, call it $\mathbf{K}^\perp$.

9. Convert $\mathbf{K}^\perp$ to an orthonormal basis $\mathbf{B}^\perp$.

10. Express the vector $\langle 1, -1, 1, -1\rangle$ as a linear combination of vectors from $\mathbf{B} \cup \mathbf{B}^\perp$.

Use the following system of equations to answer problems 11 – 14.

$$\begin{cases} 3x + 5y - 7z = 1 \\ 4x + 2y - 6z = -1 \end{cases}$$

11. Solve the homogeneous problem, and express the solution as a subspace of $\mathbb{R}^3$.

12. Solve the particular problem, and express the solution as a subspace translate of $\mathbb{R}^3$.

13. Find the equation of the plane through the origin perpendicular to your solution to problem 11 or 12.

14. Given the point $P(10, 5, -8)$ find the point on the plane closest to P, also give the distance.

Consider the linear map $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^4$ defined by $T(\langle x, y, z\rangle) = \langle x + y + z, 0, x - y + 3z, 0\rangle$ for problems 15 – 19.

15. Compute the matrix A such that $T(\overrightarrow{v}) = A\overrightarrow{v}$.

16. Find a basis for $\mathrm{Im}(T)$. What is the dimension of $\mathrm{Im}(T)$?

17. Find a basis for $\mathrm{Ker}(T)$. What is the dimension of $\mathrm{Ker}(T)$?

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18. Consider the following differential equation, given in the form $\overrightarrow{x}' = A\overrightarrow{x}$, for $\overrightarrow{x} \in \mathbb{R}^3$:

$$\overrightarrow{x}' = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & -1 \\ -8 & -5 & -3 \end{bmatrix} \overrightarrow{x}$$

(a) Find the eigenvalues and eigenvectors of A.

(b) Compute $e^{A\,t}$.

(c) Solve the ODE from this problem with the initial condition $\overrightarrow{x}(0) = \langle 1, 1, -1 \rangle$.