CS 4970 - Parallel Programming
Assignment 8 - Due 2021.11.01

Overview:

The purpose of this assignment is to perform the shooting method for solving second-order boundary-value problems.

Background:

The starting point will be the program MPI_RK2ndOrderODEEx2.cpp, which solves a second-order differential equation of the form:

$$y'' = f(x, y, y'),\ y(x_0) = y_0,\ y'(x_0) = y_0'. \tag{1}$$

Note the initial conditions for the differential equation in (1) require a known value for both $y(x)$ and $y'(x)$. Boundary-value problems are similar but instead require two initial conditions on $y(x)$:

$$y'' = f(x, y, y'),\ y(x_0) = y_0,\ y(x_1) = y_1. \tag{2}$$

We also have the following theorem:

Theorem 1. *The boundary value problem*

$$y'' = f(x, y, y'),\ y(x_0) = y_0,\ y(x_1) = y_1.$$

has a unique solution on the interval $[x_0, x_1]$ *provided that:*

(i) f *and its partial derivatives* $f_x,\ f_y,\ f_{y'}$ *are continuous on the domain* $D = [x_0, x_1] \times \mathbb{R} \times \mathbb{R}$
(ii) $f_x > 0,\ |f_y| \leq M,\ |f_{y'}| \leq M$ *on* D.

Assignment:

You are to modify the MPI_RK2ndOrderODEEx2.cpp program to perform the shooting method for solving the boundary-value problem as given in (2). The shooting method involves using equations (1) with the second condition $y'(x_0) = z$. Here z is a parameter, and by varying z one eventually locates the solution to (2) using (1).

Your program will take as input arguments the following:

(1) step-size h for the Runge-Kutta process
(2) The initial conditions for the left point $y(x_0) = y_0$, given as two separate arguments x_0 y_0
(3) The initial conditions for the right point $y(x_1) = y_1$, given as two separate arguments x_1 y_1
(4) The range of z-values $[z_0, z_1]$ for the initial condition $y'(x_0) = z$

So a typical last line of your job file may look as follows:

```
time prun ./MPI_RK2ndOrderBVP 0.001 0 1 1 2 -1.0 2.0
```

Where MPI_RK2ndOrderBVP is the name of your compiled code. For the example above, we have $h = 0.001$, $y(0) = 1$, $y(1) = 2$, and $-1 \leq y'(0) \leq 2$. Your program will break up the problem so that each process solves problems of the form given in (1) by breaking up the range of z-values $[z_0, z_1]$ in a manner predetermined by your code. The program will find the value of z such that the problem (1) also satisfies $y(x_1) = y_1$ to within a given tolerance. Use your code on the following boundary-value problems:

(1) $y''(x) - 32xy(x) = 0,\ y(0) = 0,\ y(1) = 1$
(2) $y''(x) + \dfrac{1}{1+x}y'(x) = -1,\ y(0) = 0,\ y(1) = 1$
(3) $y''(x) + \dfrac{2}{x}y'(x) - \dfrac{2}{x^2}y(x) = \dfrac{\sin(\ln(x))}{x^2},\ y(1) = 1,\ y(2) = 2$
(4) $y''(x) + (x+1)y'(x) - 2y(x) = (1 - x^2)e^{-x},\ y(0) = 0,\ y(1) = 0$