

TABLE 1a
LAGRANGE APPROXIMATIONS ON EQUALLY-SPACED GRIDS, $ch^k f^{[k]} = \sum a_j f(x_i)$

Derivative	a_{i-3}	a_{i-2}	a_{i-1}	a_i	a_{i+1}	a_{i+2}	a_{i+3}	a_{i+4}	a_{i+5}	a_{i+6}	Accuracy
$2hf_x(x_i)$			-1		1						$O(h^2)$
$2hf_x(x_i)$				-3	4	-1					$O(h^2)$
$12hf_x(x_i)$		1	-8		8	-1					$O(h^4)$
$12hf_x(x_i)$			-3	-10	18	-6	1				$O(h^4)$
$12hf_x(x_i)$				-25	48	-36	16	-3			$O(h^4)$
$60hf_x(x_i)$	-1	9	-45		45	-9	1				$O(h^6)$
$60hf_x(x_i)$		2	-24	-35	80	-30	8	-1			$O(h^6)$
$60hf_x(x_i)$			-10	-77	150	-100	50	-15	2		$O(h^6)$
$60hf_x(x_i)$				-147	360	-450	400	-225	72	-10	$O(h^6)$
$hf_x(x_{i+\frac{1}{2}})$				-1	1						$O(h^2)$
$24hf_x(x_{i+\frac{1}{2}})$			1	-27	27	-1					$O(h^4)$
$24hf_x(x_{i+\frac{1}{2}})$				-23	21	3	-1				$O(h^4)$
$1920hf_x(x_{i+\frac{1}{2}})$		-9	125	-2250	2250	-125	9				$O(h^6)$
$h^2 f_{xx}(x_i)$			1	-2	1						$O(h^2)$
$h^2 f_{xx}(x_i)$				2	-5	4	-1				$O(h^2)$
$12h^2 f_{xx}(x_i)$		-1	16	-30	16	-1					$O(h^4)$
$12h^2 f_{xx}(x_i)$			10	-15	-4	14	-6	1			$O(h^4)$
$12h^2 f_{xx}(x_i)$				35	-104	114	-56	11			$O(h^3)$
$180h^2 f_{xx}(x_i)$	2	-27	270	-490	270	-27	2				$O(h^6)$
$180h^2 f_{xx}(x_i)$		-13	228	-420	200	15	-12	2			$O(h^5)$
$180h^2 f_{xx}(x_i)$			137	-147	-255	470	-285	93	-13		$O(h^5)$
$180h^2 f_{xx}(x_i)$				812	-3132	5265	-5080	2970	-972	137	$O(h^5)$
$2h^3 f_{xxx}(x_i)$		-1	2		-2	1					$O(h^2)$
$2h^3 f_{xxx}(x_i)$			-3	10	-12	6	-1				$O(h^2)$
$2h^3 f_{xxx}(x_i)$				-5	18	-24	14	-3			$O(h^2)$
$h^4 f_{xxxx}(x_i)$		1	-4	6	-4	1					$O(h^2)$
$h^4 f_{xxxx}(x_i)$			2	-9	16	-14	6	-1			$O(h^2)$
$h^4 f_{xxxx}(x_i)$				3	-14	26	-24	11	-2		$O(h^2)$