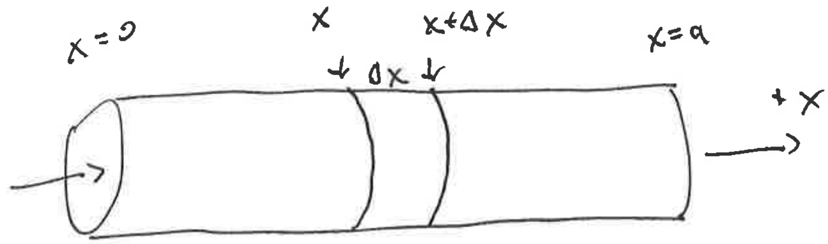

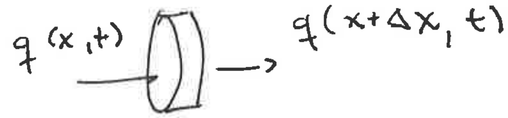

$q(x,t)$ rate of heat flow at point x at time t.

$[q] = \frac{H}{t \cdot L^2}$

H = heat
t = time
L = Length

If A is a cross-sectional area $[A] = L^2$

$[qA] = H/t$

so $A \cdot q(x,t)$ rate of heat flow into slice
$A \cdot q(x+\Delta x, t)$ rate of heat flow out of slice

Law of conservation of energy:

Amount of heat enters region + generated in region
= amount leaves the region + amount stored

Heat storage:

ρ density

c heat capacity (H/mT)

rate of heat storage:

$$\rho \cdot c \cdot A \cdot \Delta x \cdot \frac{\partial u}{\partial t}(x,t) \quad (\text{has units } H/t)$$

u is Temperature

$[\rho] = m/L^3$

$[c] = H/mT \qquad \frac{\cancel{m}}{\cancel{L^3}} \cdot \frac{H}{\cancel{mT}} \cdot \cancel{L^2} \cdot \cancel{L} \cdot \frac{\cancel{T}}{t}$

$[A] = L^2$

$[\Delta x] = L$

$\left[\frac{\partial u}{\partial t}\right] = T/t$

Heat generation – radiation / convection from surroundings
converted by electrical resistance to current
chemical / nuclear reaction

rate of generation per unit volume : g , $[g] = H/tL^3$

$$A \cdot \Delta x \, g \quad \text{has units} \quad L^2 \cdot L \cdot \frac{H}{tL^3} = H/t$$

$$\overset{\text{enter}}{A\,q(x,t)} + \overset{\text{generated}}{A\,\Delta x\,g} = \overset{\text{leaves}}{A\,q(x+\Delta x,t)} + \overset{\text{stored}}{A\,\Delta x\,\rho c\,\frac{\partial u}{\partial t}}$$

$\downarrow$

$$\frac{q(x,t) - q(x+\Delta x,t)}{\Delta x} + g = \rho c\,\frac{\partial u}{\partial t}$$

$$\lim_{\Delta x \to 0} \frac{q(x,t) - q(x+\Delta x,t)}{\Delta x} = -\frac{\partial q}{\partial x}$$

$$-\frac{\partial q}{\partial x} + g = \rho c\,\frac{\partial u}{\partial t}$$

Fourier's Law of Heat Conduction: rate of heat transfer through a material is proportional to the negative gradient in the temperature and to the area, at right angles to that gradient, through which that heat flows.

1-D answer, the x direction is how heat flows

$$q = -K\,\frac{\partial u}{\partial x}$$

Heat flows downhill ($q > 0$ when $\frac{\partial u}{\partial x} < 0$)

K is thermal conductivity constant

$$\frac{H}{t \cdot L^2} = \frac{H}{t \cdot L \cdot T}\,\frac{T}{L} \quad \text{so} \quad [K] = \frac{H}{t \cdot L \cdot T}$$

If rod not uniform, $K = K(x)$.

$$-\frac{\partial q}{\partial x} + g = \rho c \frac{\partial u}{\partial t}$$

$$\downarrow$$

$$\frac{\partial}{\partial x}\left(K \frac{\partial u}{\partial x}\right) + g = \rho c \frac{\partial u}{\partial t}$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{g}{K} = \frac{\rho c}{K} \frac{\partial u}{\partial t}$$

$$\frac{K}{\rho c} = k \quad \text{diffusivity}$$

<u>without generation:</u>

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \frac{\partial u}{\partial t}, \quad 0 < x < a, \ 0 < t$$

$$\left.\begin{aligned} u(x,t) &= x^2 + 2kt \\ u(x,t) &= e^{-kt} \sin(x) \end{aligned}\right\} \text{ are solutions} \qquad e^{-\lambda^2 kt} \begin{cases} \cos(\lambda t) \\ \sin(\lambda t) \end{cases}$$

<u>Extra conditions</u>

1. Initial temperature distribution on rod
2. What happens at ends of rod

$$u(x,0) = F(x), \quad 0 < x < a \quad \leftarrow IC$$

$$u(0,t) = T_0, \quad u(a,t) = T_1 \quad \leftarrow \begin{matrix} BC \\ t > 0 \end{matrix}$$

generally: $u(x_0,t)=\alpha(t)$, x_0 endpoint

Dirichlet condition

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heat flow rate is controlled:

$$\frac{\partial u}{\partial x}(x_0,t)=\beta(t)$$

Neumann condition

if $\beta(t)\equiv 0$ we have insulated surface

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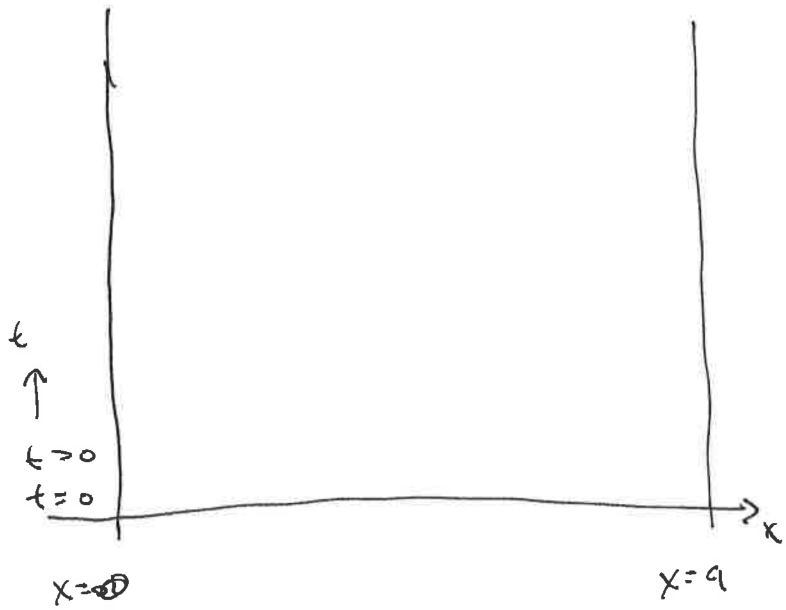
general: $c_1 u_1(x_0,t)+c_2\frac{\partial u}{\partial x}(x_0,t)=\gamma(t)$

Robin's condition


t
t > 0
t = 0
x
x = 0
x = a