

Math 3283 - Foundations of Mathematics

Exam #1 - 2021.09.13

Name: _____

The Sheffer Stroke (nand) is defined to be the negation of the conjunction, $\sim (p \wedge q)$, and is denoted by the symbol \uparrow . The fundamental truth table for $p \uparrow q$ is given by:

p	q	$p \uparrow q$
T	T	F
T	F	T
F	T	T
F	F	T

Note that by definition $p \uparrow q \Leftrightarrow \sim (p \wedge q)$. Use this definition to answer problems 1 through 3.

1. Prove by the method of truth tables that $p \uparrow p \Leftrightarrow \sim p$.
2. Prove by the method of truth tables that $p \wedge q \Leftrightarrow (p \uparrow q) \uparrow (p \uparrow q)$.
3. Through a string of equivalencies, express $p \rightarrow q$ in terms of \uparrow only.
4. Determine if each of the following propositions are true or false. Justify your answers!

(a) If $2 \cdot 3 > 10$ or $6 \cdot 3 < 20$, then $6 + 3 = 9$ and $2 + 3 = 5$.

(b) $2 \cdot 3 > 10$ iff $2 > 0$ and $3 > 0$.

5. Consider the following argument form:

$$\begin{array}{l} p \vee q \\ \sim q \\ \hline p \rightarrow r \\ \hline \therefore r \end{array}$$

(a) Verify that the argument form is valid by the method of truth tables.

(b) Using equivalencies and the simple argument forms of MP, MT, and HS, verify that the argument is valid.

6. Determine if each of the following quantified propositions are true or false. For parts (a)–(c), let U be the set of all human beings planet who have lived or are currently living. Justify your answers.

(a) $\forall x \in U \exists y \in U$ y is the mother of x

(b) $\exists x \in U \forall y \in U$ y is the mother of x

(c) $\forall x \in U \exists y \in U$ x is the mother of y

(d) $\forall x \in \mathbb{R} \exists y \in \mathbb{R}$ $y = x(x - 1)$

(e) $\forall y \in \mathbb{R} \exists x \in \mathbb{R}$ $y = x(x - 1)$

(f) $\forall y \in \mathbb{R}^+ \exists! x \in \mathbb{R}$ $y = x(x - 1)$