Math 2283 - Honors Logic

Homework - Chapter 3

Name: _____

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1. Determine which of the following sentences are true:
   (a) Oklahoma is a state in the USA.
   (b) "Oklahoma" is the name of a state in the USA.
   (c) "Oklahoma" is a state in the USA.
   (d) Oklahoma is the name of a state in the USA.

2. Prove the following law using exclusively Laws 3.3 and 3.4.
Law A: $(z = x \wedge y = z) \to x = y$

3. Prove the following using exclusively Laws 3.3 and 3.4:
Law B: $(x = y \wedge y = z \wedge t = z \wedge u = t) \to x = u$

4. Consider a triangle with sides a, b, and c. Let h_a, h_b, and h_c be the altitudes upon sides a, b, and c; similarly, let m_a, m_b, and m_c be the medians, and s_a, s_b, and s_c the bisectors of the angles of the triangle.
   (a) Assuming the triangle to be isosceles (with a as the base, and b and c as sides of equal length), which of the twelve segments named are congruent (i.e. equal in the geometric sense) and which are identical? Express the answer in the form of formulas, using the symbol "$\cong$" to designate congruence, and the symbol "$=$" to designate identity.
   (b) Solve the same problem under the assumption that the triangle is equilateral.

5. Express the following quantified expression using numerical quantifier notation:
   There are exactly three objects satisfying the given condition.

6. Determine which of the following sentences are true:
   (a) *There are at least three numbers z such that* $z^2 < 2z$.
   (b) *For any number x there is exactly one number y such that* $x + y = 2$.
   (c) *For any number x there is exactly one number y such that* $x \cdot y = 3$.

7. If, instead of considering all real numbers in Exercise 6, we consider only whole numbers (e.g. $\{0, 1, 2, \ldots\}$, does the truth value of any of the sentences change?

8. Define $P(x, y)$ to be "x is the state capital of y". Here x is a city and y is a state of the Union. Convert the following logical sentence into a numerically quantified English sentence.

$$\forall y \, \exists x \, \forall z \, [P(x, y) \wedge (P(z, y) \to (x = z))]$$

9. Determine which of the following quantified statements are true:
   (a) $\exists \, x \ (x = 1 \vee x = 2)$
   (b) $\exists \, x \ (x = 1 \wedge x = 2)$
   (c) $\exists \, x \ (x = 1 \to x = 2)$
   (d) $\exists \, x \ (x = 1 \to (x = 2 \wedge x = 1))$