

Math 2283 - Honors Logic

Homework - Chapter 4

Name: _____

1. Consider the following sets:

$$A = \{x \mid x \text{ is a US state bordering the Atlantic Ocean.}\}$$

$$B = \{x \mid x \text{ is a US state bordering the Pacific Ocean.}\}$$

$$C = \{x \mid x \text{ is a US state which has a coast on the Gulf of Mexico.}\}$$

$$D = \{x \mid x \text{ is a US state which shares a border with Canada.}\}$$

$$E = \{\text{Texas, Oklahoma, Arkansas}\}$$

$$F = \{x \mid x \text{ is a US state which shares no border with any other US state.}\}$$

$$G = \{x \mid x \text{ is a US state whose has as part of its border the Red River of the South.}\}$$

Relate the following pairs of sets using the relations: subset, superset, overlap, disjoint, and equal.

(a) A B (b) A C (c) F B

(d) E G (e) D B (f) C G

2. Determine which of the fundamental relations (cf. Theorem 4.1) hold between the following pairs of intervals:

(a) $[2, 4], [5, 8]$ (b) $[4, 7], [3, 5]$ (c) $[4, 7], [4, 7]$

(d) $[1, 3], [-2, 4]$ (e) $[1, 3], [1, 2]$ (f) $[0, 1], [-2, 1]$

(g) $[1, 7], [-2, 4]$ (h) $[-2, 10], [1, 2]$ (i) $[1, 1], [-1, 2]$

3. Draw two boxes K and L so that they stand in one of the following relations (cf. Theorem 4.1):

- (a) $K = L$
- (b) the box K is a proper subclass of the box L
- (c) the box L is a proper subclass of the box K
- (d) the boxes K and L overlap
- (e) the boxes K and L are disjoint

4. Which of the cases from problem 3 are eliminated if K and L are congruent?

5. Which of the cases from problem 3 are eliminated if we consider only the perimeters of K and L (hence K and L are rectangles)?

6. Is the following sentence (which has the same structure as the Law 4.4, the Law of Class Transitivity for Inclusion, of Section 4.4) true?

If K is disjoint from L and L is disjoint from M , then K is disjoint from M .

7. Convert the following logical sentence into set notation so that there are no logical connectives or element symbols, instead only set operations and set relations.

$$\sim (x \in K \vee x \in L) \longleftrightarrow (\sim x \in K \wedge \sim x \in L)$$

8. Let $\triangle ABC$ be an arbitrary triangle, with an arbitrary point D lying on line segment \overline{BC} . Express your answers to the following two questions in formulas:

- (a) What figures are formed by the union of the two triangles $\triangle ABD$ and $\triangle ACD$?
- (b) What figures are formed by the intersection of the two triangles $\triangle ABD$ and $\triangle ACD$?

9. Represent an arbitrary square:

- (a) as the union of two trapezoids,
- (b) as the intersection of two triangles.

10. Let K and L be two arbitrary classes. What classes are $K \cup L$ and $K \cap L$ in the case $K \subseteq L$?

11. Let K be an arbitrary class. Determine each of the following classes:

- (a) $K \cup U$ (b) $K \cap U$ (c) $\emptyset \cup K$ (d) $\emptyset \cap K$

Prove each of the following laws, given arbitrary classes K , L , and M .

12. **Law of Simplification for Union:** $K \subseteq K \cup L$

13. **Law of Simplification for Intersection:** $K \cap L \subseteq K$

14. **Distributive Law of Intersection over Union:** $K \cap (L \cup M) = (K \cap L) \cup (K \cap M)$

15. **Distributive Law of Union over Intersection:** $K \cup (L \cap M) = (K \cup L) \cap (K \cup M)$

16. **The Law of Double Complement:** $(K')' = K$

17. **De Morgan's Law for Union:** $(K \cup L)' = K' \cap L'$

18. **De Morgan's Law for Intersection:** $(K \cap L)' = K' \cup L'$

19. Consider the following three sets:

- (a) The set of all natural numbers greater than 0 and less than 4.
- (b) The set of all rational numbers greater than 0 and less than 4.
- (c) The set of all irrational numbers greater than 0 and less than 4.

Which of these set are finite and which are infinite?

20. Define $U = \mathbb{N}$ to be the set of all positive integers, and $\mathbb{K} = \{6, 12, 18, \dots\}$. Determine *all* values which make the following sentential function true.

$$\forall x \in \mathbb{K} (x/y \in \mathbb{N} \wedge y \in \mathbb{K}')$$