

# Math 2283 - Honors Logic

## Homework - Chapter 5

Name: \_\_\_\_\_

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A relation  $R$  is *co-reflexive* if and only if  $\forall x, y, xRy \rightarrow x = y$ .

A relation  $R$  is *left unique* if and only if  $\forall x_1, x_2, y, x_1Ry \wedge x_2Ry \rightarrow x_1 = x_2$ .

A relation  $R$  is *right quasi-reflexive* if and only if  $\forall x, y, xRy \rightarrow yRy$ .

1. If you look at problem 3 below, you are to prove a conditional sentence. In particular, note that in Theorem 2, you are to prove  $R$  is co-reflexive assuming  $R$  is right quasi-reflexive and left unique. The definition of co-reflexive is itself the conditional sentence  $\forall x, y, xRy \rightarrow x = y$ . One might be able to derive the entire definition of co-reflexive using only the assumptions of right quasi-reflexiveness and left uniqueness. However, it is easier if one assumes the hypothesis of the property of co-reflexiveness, and then simply derives  $x = y$  using the properties of right quasi-reflexiveness and left uniqueness. To do this, one must argue that this is indeed a valid approach. We do this in parts (a) and (b) as follows:

(a) Construct a truth table for the following sentential function:  $[(p \wedge q \wedge r) \rightarrow s] \longleftrightarrow [(p \wedge q) \rightarrow (r \rightarrow s)]$

(b) Using the result from (a), argue why the aforementioned approach to proving certain conditional theorems is valid (and can thus be used in problems 3 below).

2. Express the properties of co-reflexiveness and left uniqueness, as defined above, using ONLY the calculus of relations. For reference, see problem 9 from the Chapter 5 HW in the textbook.

3. Prove the following theorems of relations:

**Theorem 1:** If a  $R$  is asymmetrical on a class  $K$ , then the relation  $R'$  is reflexive and connected on the class  $K$ .

**Theorem 2:** If a relation  $R$  is right quasi-reflexive and left unique on a class  $K$ , then the relation  $R$  is co-reflexive on the class  $K$ .

**Theorem 3:** If a relation  $R$  is co-reflexive on a class  $K$ , then the relation  $R$  is symmetric on the class  $K$ .

4. Prove each of the following relation among relations is satisfied by arbitrary relations  $R, S$ , and  $T$ .

- (a)  $\widetilde{(R/S)} = \check{S}/\check{R}$   
(b)  $R/(S/T) = (R/S)/T$

5. Give an example of a relation which satisfies each of the following:

- (a)  $R$  is neither reflexive nor irreflexive.  
(b)  $S$  is neither symmetric nor asymmetric.  
(c)  $T$  is neither transitive nor intransitive.

6. Let  $\mathbb{R}$  be the set of all real numbers, and define the following relations:

$$xRy \longleftrightarrow x \cdot y \geq 0, \quad xSy \longleftrightarrow x - y \geq 0, \quad xTy \longleftrightarrow x - y > 0$$

- (a) Which of the relations  $R, S$ , and  $T$  are reflexive?  
(b) Which of the relations  $R, S$ , and  $T$  are transitive?  
(c) Which of the relations  $R, S$ , and  $T$  are symmetric?  
(d) Which of the relations  $R, S$ , and  $T$  are asymmetric?

(e) Which of the relations  $R$ ,  $S$ , and  $T$  are connected?

7. Attempt to define the relation of '*preceding*' with respect to two words in the English language, and then show that this relation defined an ordering on the set of all words in the English language. As a hint, consider the dictionary!

8. State the property of relations expressed by the formula:  $\check{R}/R \subseteq I$

9. Which of the relations (of the form  $xRy$ ) expressed by the following formulas are functions:

- (a)  $x$  is the mother of  $y$
- (b)  $x$  is the daughter of  $y$
- (c)  $y$  is the mother of  $x$
- (d)  $x$  is the sibling of  $y$
- (e)  $y$  is the sibling of  $x$

10. Show that the set of all natural numbers and the set of all odd numbers are equinumerous.